Numeric/Simulation

I develop a specific numerical solution of the model by using the following buyer's utility and the seller's costs functional forms:

$$U(X \mid b) = (b+1)\left(X - \frac{X^2}{2}\right) - kb$$

$$C_1(x_1 \mid \sigma) = \frac{x_1^2}{2(\sigma+1)} - \frac{\sigma^2}{2}, \text{ and } C_i(x_i) = \frac{x_i^2}{2}, \ \forall i \neq 1.$$

The efficient amount of trade is obtained when the marginal buyer's valuation is equal to the sellers' marginal production cost:

$$U_x(X \mid b) = (b+1)\left(1 - \left(x_1 + \sum_{i \neq 1}^{N} x_i\right)\right) = \frac{x_1}{\sigma+1} = C_x(x_1 \mid \sigma),$$

$$U_x(X \mid b) = (b+1)\left(1 - \left(x_1 + \sum_{i \neq 1}^{N} x_i\right)\right) = x_i = C_x(x_i), \text{ for } i \neq 1$$

Restricting attention to symmetric equilibrium, in which $x_i = x_{i'}$, for $i, i' \neq 1$, gives the explicit solution:

$$x_1^* = \frac{(\sigma+1)(b+1)}{1+(b+1)(\sigma+N)}, \tag{0.1}$$

$$x_i^* = \frac{(b+1)}{1+(b+1)(\sigma+N)}, \ for\ i \neq 1, \tag{0.2}$$

$$X^* = x_1^* + (N-1)x_i^* = \frac{(\sigma+1)(b+1)}{1+(b+1)(\sigma+N)} + \frac{(N-1)(b+1)}{1+(b+1)(\sigma+N)} = \frac{(b+1)(\sigma+N)}{1+(b+1)(\sigma+N)}. \tag{0.3}$$

These solutions, satisfy Lemma (1), i.e., $dx_1^*/d\sigma > 0$, $dx_j^*/d\sigma < 0$, $\partial X^*/\partial\sigma > 0$, and $x_i^*(1,\sigma) > x_i^*(0,\sigma)$.

$$\frac{dx_1^*}{d\sigma} = \frac{(b+1)(1+(b+1)(N-1))}{(1+(b+1)(\sigma+N))^2} > 0.$$

The allocative sensitivity is equal to:

$$\frac{dx_j^*}{d\sigma} = \frac{-(b+1)^2}{(1+(b+1)(\sigma+N))^2} < 0, \ for\ j \neq 1. \tag{0.4}$$

and the change in the aggregate amount of trade with respect to the seller's investment is:

$$\frac{dX^*}{d\sigma} = \frac{(b+1)}{(1+(b+1)(\sigma+N))^2} > 0.$$

Moreover, trade from each seller increases when the buyer invests:

$$x_1^*(1,\sigma) = \frac{2(\sigma+1)}{1+2(\sigma+N)} > \frac{\sigma+1}{1+\sigma+N} = x_1^*(0,\sigma) \iff 1 > 0.$$

$$x_i^*(1,\sigma) = \frac{2}{1+2(\sigma+N)} > \frac{1}{1+\sigma+N} = x_1^*(0,\sigma) \iff 1 > 0, \; for \; i \neq 1.$$

1. Seller's investment

In the most intense competition, the investment decision rule is efficient $\psi_\sigma(\sigma) = -C_\sigma(x_1^*(\sigma,b) \mid \sigma)$, and σ_b^* is implicitly defined by:

$$\sigma_b^* = \frac{\left(\frac{(\sigma_b^*+1)(b+1)}{1+(b+1)(\sigma_b^*+N)}\right)^2}{2(1+\sigma_b^*)^2} = \frac{(b+1)^2}{2\left(1+(b+1)(\sigma_b^*+N)\right)^2}. \tag{1.1}$$

When competition is less intense, the seller's investing decision rule becomes:

$$\psi_\sigma(\sigma) = -C_\sigma(x_1^*(\sigma,b) \mid \sigma) - \frac{\partial \tilde{TS}_{-1}(b,\sigma \mid J_1)}{\partial \sigma}.$$

with

$$\frac{\partial \tilde{TS}_{-1}(b,\sigma \mid J_1)}{\partial \sigma} = [N-(1+J_1)]\left[U_x\left(X^*_{-\{J_1,1\}} + \sum_{j\in J_1}\tilde{x}_j\right) - U_x(X^*)\right]\frac{dx_l^*}{d\sigma}$$

where sellers $l \subset N$ do not offer latent contracts. To obtain the input $\tilde{x}_j$ in the latent contracts for any $j \in J_1$, I make use of the optimal condition:

$$U_x\left(X^*_{-\{J_1,1\}} + \sum_{j\in J_1}\tilde{x}_j\right) = C_x(x_j^*)$$

Operating, and restricting attention to symmetric equilibrium, I obtain:

$$(b+1)\left(1-\left(X^*_{-\{J_1,1\}} + J_1\tilde{x}_j\right)\right) = \tilde{x}_j \Longrightarrow (b+1)\left(1-\left(\frac{(N-(J_1+1))\,(b+1)}{1+(b+1)(\sigma+N)} + J_1\tilde{x}_j\right)\right) = \tilde{x}_j$$

which gives and explicit solution

$$\tilde{x}_j = \frac{(b+1)\left[1+(b+1)(\sigma+J_1+1)\right]}{(1+(b+1)(\sigma+N))(1+(b+1)J_1)} \tag{1.2}$$

and

$$\begin{aligned} X^*_{-\{J_1,1\}} + \sum_{j\in J_1}\tilde{x}_j &= \frac{(N-(J_1+1))\,(b+1)}{1+(b+1)(\sigma+N)} + \frac{J_1(b+1)\left[1+(b+1)(\sigma+J_1+1)\right]}{(1+(b+1)(\sigma+N))(1+(b+1)J_1)} \\ &= \frac{(b+1)\left[N+(b+1)J_1(N+\sigma)-1\right]}{(1+(b+1)(\sigma+N))(1+(b+1)J_1)}, \end{aligned} \tag{1.3}$$

where $X^*_{-\{J_1,1\}}$ is obtained multiplying $(N-(J_1+1))$ to expression (0.2). This solution satisfies Lemma (2) (not trading with any seller i generates a smaller aggregate trade) $X^* > X^*_{-\{J_1,1\}} + \sum_{j\in J_1}\tilde{x}_j$,

$$X^* = \frac{(b+1)(\sigma+N)}{1+(b+1)(\sigma+N)} > \frac{(b+1)\left[N+(b+1)J_1(N+\sigma)-1\right]}{(1+(b+1)(\sigma+N))(1+(b+1)J_1)} = X^*_{-\{J_1,1\}} + \sum_{j\in J_1}\tilde{x}_j$$

$$\iff (\sigma+N)(1+(b+1)(\sigma+N)) > N+(b+1)J_1(N+\sigma)-1 \iff \sigma > -1.$$

The solution for $\tilde{x}_j$ satisfies Lemma (3) (the input offered in the latent contracts is non-increasing with the number of sellers who offer latent contracts). To see this, I use a continuous approximation for the set J_1 to obtain:

$$\frac{\partial \tilde{x}_j}{\partial J_1} = \frac{-(b+1)^3\left((\sigma+1)(1+(b+1)(\sigma+N))\right)}{\left[(1+(b+1)(\sigma+N))(1+(b+1)J_1)\right]^2} < 0.$$

Introducing $X^*_{-\{J_1,1\}} + \sum_{j\in J_1}\tilde{x}_j$, X^* and $dx^*_l/d\sigma$ into expression $(\partial \tilde{TS}_{-1}(b,\sigma \mid J_1))/\partial\sigma$, I get:

$$\begin{aligned}\frac{\partial \tilde{TS}_{-1}(b,\sigma \mid J_1)}{\partial\sigma} &= (N-(J_1+1))\left[\frac{(b+1)^2(\sigma+N)}{1+(b+1)(\sigma+N)} - \frac{(b+1)^2\left[N(1+(b+1)J_1)+J_1(b+1)\sigma-1\right]}{(1+(b+1)(\sigma+N))(1+(b+1)J_1)}\right]\frac{dx^*_l}{d\sigma}\\ &= -(N-(J_1+1))\left[\frac{(b+1)^2(\sigma+1)}{(1+(b+1)(\sigma+N))(1+(b+1)J_1)}\right]\frac{(b+1)^2}{(1+(b+1)(\sigma+N))^2}\\ &= -\frac{(b+1)^4(N-(J_1+1))(\sigma+1)}{(1+(b+1)(\sigma+N))^2(1+(b+1)J_1)}.\end{aligned}$$

Then, the equilibrium investment $\hat{\sigma}_b(J_1)$ is implicitly characterized by:

$$\begin{aligned}\hat{\sigma}_b(J_1) &= \frac{(b+1)^2}{2\left(1+(b+1)(\hat{\sigma}_b(J_1)+N)\right)^2} + \frac{(b+1)^4(N-(J_1+1))(\hat{\sigma}_b(J_1)+1)}{(1+(b+1)(\hat{\sigma}_b(J_1)+N))^2(1+(b+1)J_1)}\\ &= \frac{(b+1)^2\left[1+(b+1)J_1+2(b+1)^2\left((N-(J_1+1))(\hat{\sigma}_b(J_1)+1)\right)\right]}{2(1+(b+1)J_1)(1+(b+1)(\hat{\sigma}_b(J_1)+N))^2}.\end{aligned} \tag{1.4}$$

Note that for $J_1 = N-1$ (the most intense competition), expression (1.4) coincides with (1.1). Because the function (1.4) is in implicit form, I make use of Matlab to obtain the solution of $\hat{\sigma}(J_1)$ for any set $J_1 \subset N$ (Matlab codes can be found at the end of the document). The following table gives the seller's efficient and equilibrium investment for any $J_1 \subset N$, and with $N = 10$ sellers.

$N=10$	$b=0$	$b=1$
σ^*_b	0.003754	0.007171
$\hat{\sigma}_b(J_1=9)$	0.003754	0.007171
$\hat{\sigma}_b(J_1=8)$	0.004545	0.010321
$\hat{\sigma}_b(J_1=7)$	0.005514	0.014123
$\hat{\sigma}_b(J_1=6)$	0.006727	0.018804
$\hat{\sigma}_b(J_1=5)$	0.008291	0.024708
$\hat{\sigma}_b(J_1=4)$	0.010382	0.032385
$\hat{\sigma}_b(J_1=3)$	0.013323	0.042775
$\hat{\sigma}_b(J_1=2)$	0.017761	0.057621
$\hat{\sigma}_b(J_1=1)$	0.025234	0.080569

2. Buyer's investment

With regards to the buyer's investment decision, he invests if the fixed cost of investment stay below the gains from the investment that he can appropriate. Then, the buyer's efficient investment decision is to invest whenever $k \leq K^*$, and not to invest otherwise. The efficient buyer's investment threshold is:

$$K^* \equiv TS^*(1, \sigma_1^*) - TS^*(0, \sigma_0^*) - (\psi(\sigma_1^*) - \psi(\sigma_0^*)),$$

where

$$\begin{aligned}
TS^*(1, \sigma_1^*) &= 2\left(X^*(1, \sigma_1^*) - (X^*(1, \sigma_1^*))^2/2\right) - \frac{(x_1^*(1, \sigma_1^*))^2}{2(\sigma_1^* + 1)} - (N-1)\frac{(x_i^*(1, \sigma_1^*))^2}{2} \\
&= 2\left(\frac{2(\sigma_1^* + N)}{1 + 2(\sigma_1^* + N)} - \frac{\left(\frac{2(\sigma_1^*+N)}{1+2(\sigma_1^*+N)}\right)^2}{2}\right) - \frac{\left(\frac{2(\sigma_1^*+1)}{1+2(\sigma_1^*+N)}\right)^2}{2(\sigma_1^* + 1)} - (N-1)\frac{\left(\frac{2}{1+2(\sigma_1^*+N)}\right)^2}{2} \\
&= 2\left(\frac{2(\sigma_1^* + N)(1 + 2(\sigma_1^* + N)) - 2(\sigma_1^* + N)^2}{(1 + 2(\sigma_1^* + N))^2}\right) - \frac{2(\sigma_1^* + N)}{(1 + 2(\sigma_1^* + N))^2} \\
&= 2\left(\frac{2(\sigma_1^* + N)(1 + \sigma_1^* + N)}{(1 + 2(\sigma_1^* + N))^2}\right) - \frac{2(\sigma_1^* + N)}{(1 + 2(\sigma_1^* + N))^2} \\
&= \frac{2(\sigma_1^* + N)}{1 + 2(\sigma_1^* + N)},
\end{aligned}$$

$$\begin{aligned}
TS^*(0, \sigma_0^*) &= \left(X^*(0, \sigma_0^*) - (X^*(0, \sigma_0^*))^2/2\right) - \frac{(x_1^*(0, \sigma_0^*))^2}{2(\sigma_0^* + 1)} - (N-1)\frac{(x_i^*(0, \sigma_0^*))^2}{2} \\
&= \left(\frac{(\sigma_0^* + N)}{1 + (\sigma_0^* + N)} - \frac{\left(\frac{(\sigma_0^*+N)}{1+(\sigma_0^*+N)}\right)^2}{2}\right) - \frac{\left(\frac{(\sigma_0^*+1)}{1+(\sigma_0^*+N)}\right)^2}{2(\sigma_0^* + 1)} - (N-1)\frac{\left(\frac{1}{1+(\sigma_0^*+N)}\right)^2}{2} \\
&= \frac{2(\sigma_0^* + N)(1 + \sigma_0^* + N) - (\sigma_0^* + N)^2}{2(1 + \sigma_0^* + N)^2} - \frac{\sigma_0^* + N}{2(1 + \sigma_0^* + N)^2} \\
&= \frac{(\sigma_0^* + N)(2 + \sigma_0^* + N)}{2(1 + \sigma_0^* + N)^2} - \frac{\sigma_0^* + N}{2(1 + \sigma_0^* + N)^2} \\
&= \frac{\sigma_0^* + N}{2(1 + \sigma_0^* + N)},
\end{aligned}$$

and

$$(\psi(\sigma_1^*) - \psi(\sigma_0^*)) = \left(\frac{(\sigma_1^*)^2}{2} - \frac{(\sigma_0^*)^2}{2}\right).$$

Collecting terms gives:

$$K^* = \frac{2(\sigma_1^* + N)}{1 + 2(\sigma_1^* + N)} - \frac{(\sigma_0^* + N)}{2(1 + \sigma_0^* + N)} - \frac{1}{2}(\sigma_1^* + \sigma_0^*)(\sigma_1^* - \sigma_0^*).$$

In equilibrium, the buyer invests if $k \leq \hat{K}(J)$ and decides not to invest otherwise. The buyer's equilibrium investment threshold is:

$$\begin{aligned}
\hat{K}(J) &\equiv TS^*(1,\hat{\sigma}_1(J_1)) - TS^*(0,\hat{\sigma}_0(J_1)) \\
&\quad - \sum_{i\in N} \left[(T_i^*(1,\hat{\sigma}_1(J_1) \mid J_i) - C_i(x_i^*(1,\hat{\sigma}_1(J_1)))) - (T_i^*(0,\hat{\sigma}_0(J_1)|J_i) - C_i(x_i^*(0,\hat{\sigma}_0(J_1))))\right] \\
&= -(N-1)\left(TS^*(1,\hat{\sigma}_1(J_1)) - TS^*(0,\hat{\sigma}_0(J_1))\right) + \left(\tilde{TS}_{-1}(1,\hat{\sigma}_1(J_1) \mid J_1) - \tilde{TS}_{-1}(0,\hat{\sigma}_0(J_1) \mid J_1)\right) \\
&\quad + (N-1)\left(\tilde{TS}_{-i}(1,\hat{\sigma}_1(J_1) \mid J_i) - \tilde{TS}_{-i}(0,\hat{\sigma}_0(J_1) \mid J_i)\right)
\end{aligned}$$

For the calculation of the investing threshold of $\hat{K}(J)$, I simplify each of the elements. Then:

$$-(N-1)TS^*(1,\hat{\sigma}_1(J_1) = -\frac{2(N-1)(\hat{\sigma}_1(J_1)+N)}{1+2(\hat{\sigma}_1(J_1)+N)}.$$

$$(N-1)TS^*(0,\hat{\sigma}_0(J_1)) = \frac{(N-1)(\hat{\sigma}_0(J_1)+N)}{2(1+\hat{\sigma}_0(J_1)+N)}.$$

$$\begin{aligned}
&\tilde{TS}_{-1}(1,\hat{\sigma}_1(J_1) \mid J_1) \\
&= 2\left((X^*_{-\{J_1,1\}}(1,\hat{\sigma}_1(J_1)) + \sum_{j\in J_1} \tilde{x}_j(1,\hat{\sigma}_1(J_1)) - \frac{\left(X^*_{-\{J_1,1\}}(1,\hat{\sigma}_1(J_1)) + \sum_{j\in J_1} \tilde{x}_j(1,\hat{\sigma}_1(J_1))\right)^2}{2} \right) \\
&\quad - \sum_{i\neq\{J_1,1\}} C_i(x_i^*(1,\hat{\sigma}_1(J_1))) - \sum_{j\in J_1} C_j(\tilde{x}_j(1,\hat{\sigma}_1(J_1))) \\
&= 2\left(\frac{2\left[N+2J_1(N+\hat{\sigma}_1(J_1))-1\right]}{(1+2(\hat{\sigma}_1(J_1)+N))(1+2J_1)} - \frac{\left(\frac{2[N+2J_1(N+\hat{\sigma}_1(J_1))-1]}{(1+2(\hat{\sigma}_1(J_1)+N))(1+2J_1)}\right)^2}{2} \right) \\
&\quad - (N-(J_1+1))\frac{\left(\frac{2}{1+2(N+\hat{\sigma}_1(J_1))}\right)^2}{2} - J_1\frac{\left(\frac{2[1+2(\hat{\sigma}_1(J_1)+J_1+1)]}{(1+2(\hat{\sigma}_1(J_1)+N))(1+2J_1)}\right)^2}{2} \\
&= 2\left(\frac{2\left[N+2J_1(N+\hat{\sigma}_1(J_1))-1\right]}{(1+2(\hat{\sigma}_1(J_1)+N))(1+2J_1)} - \frac{2\left[N+2J_1(N+\hat{\sigma}_1(J_1))-1\right]^2}{(1+2(\hat{\sigma}_1(J_1)+N))^2(1+2J_1)^2} \right) - (N-J_1-1)\frac{2}{(1+2(\hat{\sigma}_1(J_1)+N))^2} \\
&\quad - J_1\frac{2\left[1+2(\hat{\sigma}_1(J_1)+J_1+1)\right]^2}{(1+2(\hat{\sigma}_1(J_1)(J_1+N))^2(1+2J_1)^2} \\
&= \frac{4\left[N+2J_1(N+\hat{\sigma}_1(J_1)-1\right]}{(1+2(\hat{\sigma}_1(J_1)+N))(1+2J_1)}\left(\frac{(1+2(\hat{\sigma}_1(J_1)(J_1+N))(1+2J_1) - \left[N+2J_1(N+\hat{\sigma}_1(J_1))-1\right]}{(1+2(\hat{\sigma}_1(J_1)+N))(1+2J_1)}\right) \\
&\quad - (N-J_1-1)\frac{2}{(1+2(\hat{\sigma}_1(J_1)+N))^2} - J_1\frac{2\left[1+2(\hat{\sigma}_1(J_1)+J_1+1)\right]^2}{(1+2(\hat{\sigma}_1(J_1)+N))^2(1+2J_1)^2} \\
&= \frac{4\left[N+2J_1(N+\hat{\sigma}_1(J_1))-1\right]}{(1+2(\hat{\sigma}_1(J_1)+N))(1+2J_1)}\left(\frac{2(1+J_1)(1+\hat{\sigma}_1(J_1))+N(1+2J_1)}{(1+2(\hat{\sigma}_1(J_1)+N))(1+2J_1)}\right) - (N-J_1-1)\frac{2}{(1+2(\hat{\sigma}_1(J_1)+N))^2} \\
&\quad - J_1\frac{2\left[1+2(\hat{\sigma}_1(J_1)+J_1+1)\right]^2}{(1+2(\hat{\sigma}_1(J_1)+N))^2(1+2J_1)^2}.
\end{aligned}$$

$$\tilde{TS}_{-1}(0,\hat{\sigma}_0(J_1)\mid J_1)=$$

$$2\left((X^*_{-\{J_1,1\}}(0,\hat{\sigma}_0(J_1))+\sum_{j\in J_1}\tilde{x}_j(0,\hat{\sigma}_0(J_1))-\frac{\left(X^*_{-\{J_1,1\}}(0,\hat{\sigma}_0(J_1))+\sum_{j\in J_1}\tilde{x}_j(0,\hat{\sigma}_0(J_1)\right)^2}{2}\right)$$

$$-\sum_{i\neq\{J_1,1\}}C_i(x_i^*(0,\hat{\sigma}_0(J_1)))-\sum_{j\in J_1}C_j(\tilde{x}_j(0,\hat{\sigma}_0(J_1)))$$

$$=\left(\frac{[N+J_1(N+\hat{\sigma}_0(J_1))-1]}{(1+(\hat{\sigma}_0(J_1)+N))(1+J_1)}-\frac{\left(\frac{[N+J_1(N+\hat{\sigma}_0(J_1))-1]}{(1+(\hat{\sigma}_0(J_1)+N))(1+J_1)}\right)^2}{2}\right)-(N-(J_1+1))\frac{\left(\frac{1}{1+(1+\hat{\sigma}_0(J_1))}\right)^2}{2}$$

$$-J_1\frac{\left(\frac{[1+(\hat{\sigma}_0(J_1)+J_1+1)]}{(1+(\hat{\sigma}_0(J_1)+N))(1+J_1)}\right)^2}{2}$$

$$=-\left(\frac{[N+J_1(N+\hat{\sigma}_0(J_1))-1]}{(1+\hat{\sigma}_0(J_1)+N)(1+J_1)}-\frac{[N+J_1(N+\hat{\sigma}_0(J_1))-1]^2}{2(1+\hat{\sigma}_0(J_1)+N)^2(1+J_1)^2}\right)+(N-J_1-1)\frac{1}{2(1+\hat{\sigma}_0(J_1)+N)^2}$$

$$+J_1\frac{[2+\hat{\sigma}_0(J_1)+J_1]^2}{2(1+\hat{\sigma}_0(J_1)+N)^2(1+J_1)^2}$$

$$=-\frac{[N+J_1(N+\hat{\sigma}_0(J_1))-1]}{(1+\hat{\sigma}_0(J_1)+N)(1+J_1)}\left(\frac{2(1+\hat{\sigma}_0(J_1)+N)(1+J_1)-[N+J_1(N+\hat{\sigma}_0(J_1))-1]}{2(1+\hat{\sigma}_0(J_1)+N)(1+J_1)}\right)$$

$$+\frac{1}{2(1+\hat{\sigma}_0(J_1)+N)^2}\left[N-J_1-1+J_1\frac{[2+\hat{\sigma}_0(J_1)+J_1]^2}{(1+J_1)^2}\right]$$

$$=-\frac{[N+J_1(N+\hat{\sigma}_0(J_1))-1]}{(1+\hat{\sigma}_0(J_1)+N)(1+J_1)}\left(\frac{1+\hat{\sigma}_0(J_1)+(2+\hat{\sigma}_0(J_1)+N)(1+J_1)}{2(1+\hat{\sigma}_0(J_1)+N)(1+J_1)}\right)+\frac{1}{2(1+\hat{\sigma}_0(J_1)+N)^2}\left[N-J_1-1\right.$$

$$\left.+J_1\frac{[2+\hat{\sigma}_0(J_1)+J_1]^2}{(1+J_1)^2}\right].$$

Where the trading quantities $\tilde{x}_j$ and $\left(X^*_{-\{J_1,1\}}+\sum_{j\in J_1}\tilde{x}_j\right)$ in the third and fourth line of $\tilde{TS}_{-1}(1,\hat{\sigma}_1(J_1)\mid J_1)$ and $\tilde{TS}_{-1}(0,\hat{\sigma}_0(J_1)\mid J_1)$ are obtained from expressions (1.2) and (1.3) respectively. For the calculations of $TS_{-i}(b,\sigma\mid J_i)$, note that:

$$\tilde{TS}_{-i}(b,\sigma\mid J_i)=U\left(X^*_{-\{J_i,i\}}+\tilde{x}_1+\sum_{j\in J_i\setminus\{1\}}\tilde{x}_j\right)-\sum_{l\in N\setminus\{J_i,i\}}C_l(x_l^*)-C_1(\tilde{x}_1)-\sum_{j\in J_i\setminus\{1\}}C_j(\tilde{x}_j)$$

To obtain the quantities $\tilde{x}_1$ and $\tilde{x}_j$, I use the optimal conditions:

$$U_x\left(X^*_{-\{J_i,i\}}+\tilde{x}_1+\sum_{j\in J_i\setminus\{1\}}\tilde{x}_j\right)=\frac{\tilde{x}_1}{\sigma+1}=C_x(\tilde{x}_1\mid\sigma),$$
$$U_x\left(X^*_{-\{J_i,i\}}+\tilde{x}_1+\sum_{j\in J_i\setminus\{1\}}\tilde{x}_j\right)=\tilde{x}_j=C_x(\tilde{x}_j)$$

Using the functional form for $U(X\mid b)$ and simple algebra gives:

$$\tilde{x}_1=\frac{(\sigma+1)(b+1)\left[1+(b+1)(\sigma+J_i+1)\right]}{(1+(b+1)(\sigma+N))(1+(b+1)(J_i+\sigma)}\tag{2.1}$$

and

$$\tilde{x}_j = \frac{(b+1)\left[1+(b+1)(\sigma+J_i+1)\right]}{(1+(b+1)(\sigma+N))(1+(b+1)(J_i+\sigma)}. \tag{2.2}$$

Then,

$$\begin{aligned} X^*_{-\{J_i,i\}} + \tilde{x}_1 + \sum_{j\in J_i\setminus\{1\}} \tilde{x}_j &= (N-(J_i+1))\frac{b+1}{(1+(b+1)(\sigma+N)} + \frac{(\sigma+1)(b+1)\left[1+(b+1)(\sigma+J_i+1)\right]}{(1+(b+1)(\sigma+N))(1+(b+1)(J_i+\sigma)} \\ &+ (J_i-1)\frac{(b+1)\left[1+(b+1)(\sigma+J_i+1)\right]}{(1+(b+1)(\sigma+N))(1+(b+1)(J_i+\sigma)} = \frac{(b+1)\left[1+(b+1)(\sigma+J_i)(\sigma+N)\right]}{(1+(b+1)(\sigma+N))(1+(b+1)(J_i+\sigma)} \end{aligned} \tag{2.3}$$

This solution satisfies Lemma (2) because $X^* > X^*_{-\{J_i,i\}} + \tilde{x}_1 + \sum_{j\in J_i\setminus\{1\}}\tilde{x}_j$,

$$X^* = \frac{(b+1))(\sigma+N)}{1+(b+1)(\sigma+N)} > \frac{(b+1)\left[1+(b+1)(\sigma+J_i)(\sigma+N)\right]}{(1+(b+1)(\sigma+N))(1+(b+1)(J_i+\sigma)} = X^*_{-\{J_i,i\}} + \tilde{x}_1 + \sum_{j\in J_i\setminus\{1\}}\tilde{x}_j$$

$$\iff (\sigma+N)(1+(b+1)(\sigma+J_i)) > 1+(b+1)(\sigma+J_i)(\sigma+N) \iff \sigma+(N-1)>0.$$

Introducing the result of (2.3) into $\tilde{TS}_{-i}(b,\sigma \mid J_i)$ gives:

$$\begin{aligned} &(N-1)\tilde{TS}_{-i}(1,\hat{\sigma}_1(J_1)\mid J_i) = \\ &2(N-1)\left(\frac{2\left[1+2(\hat{\sigma}_1(J_1)+J_i)(\hat{\sigma}_1(J_1)+N)\right]}{(1+2(\hat{\sigma}_1(J_1)+N))(1+2(J_i+\hat{\sigma}_1(J_1))} - \frac{\left(\frac{2[1+2(\hat{\sigma}_1(J_1)+J_i)(\hat{\sigma}_1(J_1)+N)]}{(1+2(\hat{\sigma}_1(J_1)+N))(1+2(J_i+\hat{\sigma}_1(J_1))}\right)^2}{2}\right) \\ &-(N-1)(N-(J_i+1))\frac{\left(\frac{2}{(1+2(\hat{\sigma}_1(J_1)+N)}\right)^2}{2} - (N-1)\frac{\left(\frac{(\hat{\sigma}_1(J_1)+1)2[1+2(\hat{\sigma}_1(J_1)+J_i+1)]}{(1+2(\hat{\sigma}_1(J_1)+N))(1+2(J_i+\hat{\sigma}_1(J_1))}\right)^2}{2} \\ &-(N-1)(J_i-1)\frac{\left(\frac{2[1+2(\hat{\sigma}_1(J_1)+J_i+1)]}{(1+2(\hat{\sigma}_1(J_1)+N))(1+2(J_i+\hat{\sigma}_1(J_1))}\right)^2}{2} \\ &=(N-1)\left[\frac{4\left[1+2(\hat{\sigma}_1(J_1)+J_i)(\hat{\sigma}_1(J_1)+N)\right]}{(1+2(\hat{\sigma}_1(J_1)+N))(1+2(J_i+\hat{\sigma}_1(J_1)))} - \frac{4\left[1+2(\hat{\sigma}_1(J_1)+J_i)(\hat{\sigma}_1(J_1)+N)\right]^2}{(1+2(\hat{\sigma}_1(J_1)+N))^2(1+2(J_i+\hat{\sigma}_1(J_1)))^2}\right] \\ &-(N-1)(N-J_i-1)\frac{2}{(1+2(\hat{\sigma}_1(J_1)+N))^2} - (N-1)\frac{2(\hat{\sigma}_1(J_1)+1)^2\left[1+2(\hat{\sigma}_1(J_1)+J_i+1)\right]^2}{(1+2(\hat{\sigma}_1(J_1)+N))^2(1+2(J_i+\hat{\sigma}_1(J_1)))^2} \\ &-(N-1)(J_i-1)\frac{2\left[1+2(\hat{\sigma}_1(J_1)+J_i+1)\right]^2}{(1+2(\hat{\sigma}_1(J_1)+N))^2(1+2(J_i+\hat{\sigma}_1(J_1)))^2} \\ &=(N-1)\frac{4\left[1+2(\hat{\sigma}_1(J_1)+J_i)(\hat{\sigma}_1(J_1)+N)\right]}{(1+2(\hat{\sigma}_1(J_1)+N))(1+2(J_i+\hat{\sigma}_1(J_1)))}\left[\frac{2(\hat{\sigma}_1(J_1)+J_i)+2(\hat{\sigma}_1(J_1)+N)(1+\hat{\sigma}_1(J_1)+J_i)}{(1+2(\hat{\sigma}_1(J_1)+N))(1+2(J_i+\hat{\sigma}_1(J_1)))}\right] \\ &-(N-1)(N-J_i-1)\frac{2}{(1+2(\hat{\sigma}_1(J_1)+N))^2} \\ &-\frac{2(N-1)\left[1+2(\hat{\sigma}_1(J_1)+J_i+1)\right]^2}{(1+2(\hat{\sigma}_1(J_1)+N))^2(1+2(J_i+\hat{\sigma}_1(J_1)))^2}\left[2(\hat{\sigma}_1(J_1)+1)^2+J_i-1\right]. \end{aligned}$$

$$\tilde{TS}_{-i}(0, \hat{\sigma}_0(J_1) \mid J_i) =$$

$$\left(\frac{[1 + (\hat{\sigma}_0(J_1) + J_i)(\hat{\sigma}_0(J_1) + N)]}{(1 + (\hat{\sigma}_0(J_1) + N))(1 + (J_i + \hat{\sigma}_0(J_1))} - \frac{\left(\frac{[1+(\hat{\sigma}_0(J_1)+J_i)(\hat{\sigma}_0(J_1)+N)]}{(1+(\hat{\sigma}_0(J_1)+N))(1+(J_i+\hat{\sigma}_0(J_1))}\right)^2}{2} \right)$$

$$- (N - (J_i + 1)) \frac{\left(\frac{1}{(1+(\hat{\sigma}_0(J_1)+N)}\right)^2}{2} - \frac{\left(\frac{(\hat{\sigma}_0(J_1)+1)[1+(\hat{\sigma}_0(J_1)+J_i+1)]}{(1+(\hat{\sigma}_0(J_1)+N))(1+(J_i+\hat{\sigma}_0(J_1))}\right)^2}{2}$$

$$- (J_i - 1) \frac{\left(\frac{[1+(\hat{\sigma}_0(J_1)+J_i+1)]}{(1+(\hat{\sigma}_0(J_1)+N))(1+(J_i+\hat{\sigma}_0(J_1))}\right)^2}{2}$$

$$= -(N-1) \left(\frac{[1 + (\hat{\sigma}_0(J_1) + J_i)(\hat{\sigma}_0(J_1) + N)]}{(1 + \hat{\sigma}_0(J_1) + N)(1 + J_i + \hat{\sigma}_0(J_1))} - \frac{[1 + (\hat{\sigma}_0(J_1) + J_i)(\hat{\sigma}_0(J_1) + N)]^2}{2(1 + \hat{\sigma}_0(J_1) + N)^2(1 + J_i + \hat{\sigma}_0(J_1))^2} \right)$$

$$+ (N-1)(N - J_i - 1) \frac{1}{2(1 + \hat{\sigma}_0(J_1) + N)^2} + (N-1) \frac{(\hat{\sigma}_0(J_1) + 1)^2 \left[2 + \hat{\sigma}_0(J_1) + J_i\right]^2}{2(1 + \hat{\sigma}_0(J_1) + N)^2(1 + J_i + \hat{\sigma}_0(J_1))^2}$$

$$+ (N-1)(J_i - 1) \frac{\left[2 + \hat{\sigma}_0(J_1) + J_i\right]^2}{2(1 + \hat{\sigma}_0(J_1) + N)^2(1 + J_i + \hat{\sigma}_0(J_1))^2}$$

$$= -(N-1) \frac{[1 + (\hat{\sigma}_0(J_1) + J_i)(\hat{\sigma}_0(J_1) + N)]}{(1 + \hat{\sigma}_0(J_1) + N)(1 + J_i + \hat{\sigma}_0(J_1))} \left(\frac{1 + 4\hat{\sigma}_0(J_1) + 2(J_i + N) + (\hat{\sigma}_0(J_1) + J_i)(\hat{\sigma}_0(J_1) + N)}{2(1 + \hat{\sigma}_0(J_1) + N)(1 + J_i + \hat{\sigma}_0(J_1))} \right)$$

$$+ (N-1)(N - J_i - 1) \frac{1}{2(1 + \hat{\sigma}_0(J_1) + N)^2} + \frac{(N-1)\left(2 + \hat{\sigma}_0(J_1) + J_i\right)^2}{2(1 + \hat{\sigma}_0(J_1) + N)^2(1 + J_i + \hat{\sigma}_0(J_1))^2} \left[(\hat{\sigma}_0(J_1) + 1)^2 + J_i - 1\right]$$

Where the trading quantities $\tilde{x}_1$, $\tilde{x}_j$ and $X^*_{-\{J_i, i\}} + \tilde{x}_1 + \sum_{j \in J_i \setminus \{1\}} \tilde{x}_j$ for $\tilde{TS}_{-i}(1, \hat{\sigma}_1(J_1) \mid J_i)$ and $\tilde{TS}_{-i}(0, \hat{\sigma}_0(J_i) \mid J_1)$ are obtained from expressions (2.1),(2.2) and (2.3) respectively. Collecting all the previous terms, the buyer's equilibrium investment threshold $\hat{K}(J)$ becomes:

$$\hat{K}(J) = -\frac{2(N+1)(\hat{\sigma}_1(J_1) + N)}{1 + 2(\hat{\sigma}_1(J_1) + N)} + \frac{(N-1)(\hat{\sigma}_0(J_1) + N)}{2(1 + \hat{\sigma}_0(J_1) + N)}$$

$$+ \frac{4\left[N + 2J_1(N + \hat{\sigma}_1(J_1)) - 1\right]}{(1 + 2(\hat{\sigma}_1(J_1) + N))(1 + 2J_1)} \left(\frac{2(1 + J_1)(1 + \hat{\sigma}_1(J_1)) + N(1 + 2J_1)}{(1 + 2(\hat{\sigma}_1(J_1) + N))(1 + 2J_1)} \right) - (N - J_1 - 1) \frac{2}{(1 + 2(\hat{\sigma}_1(J_1) + N))^2}$$

$$- J_1 \frac{2\left[1 + 2(\hat{\sigma}_1(J_1) + J_1 + 1)\right]^2}{(1 + 2(\hat{\sigma}_1(J_1) + N))^2(1 + 2J_1)^2} + \frac{1}{2(1 + \hat{\sigma}_0(J_1) + N)^2} \left[N - J_1 - 1 + J_1 \frac{\left[2 + \hat{\sigma}_0(J_1) + J_1\right]^2}{(1 + J_1)^2} \right]$$

$$- \frac{\left[N + J_1(N + \hat{\sigma}_0(J_1)) - 1\right]}{(1 + \hat{\sigma}_0 + N)(1 + J_1)} \left(\frac{1 + \hat{\sigma}_0(J_1) + (2 + \hat{\sigma}_0(J_1) + N)(1 + J_1)}{2(1 + \hat{\sigma}_0(J_1) + N)(1 + J_1)} \right) + \frac{1}{2(1 + \hat{\sigma}_0(J_1) + N)^2} \left[N - J_1 - 1 \right.$$

$$\left. + J_1 \frac{\left[2 + \hat{\sigma}_0(J_1) + J_1\right]^2}{(1 + J_1)^2} \right]$$

$$+ (N-1) \frac{4\left[1 + 2(\hat{\sigma}_1(J_1) + J_i)(\hat{\sigma}_1(J_1) + N)\right]}{(1 + 2(\hat{\sigma}_1(J_1) + N))(1 + 2(J_i + \hat{\sigma}_1(J_1)))} \left[\frac{2(\hat{\sigma}_1(J_1) + J_i) + 2(\hat{\sigma}_1(J_1) + N)(1 + \hat{\sigma}_1(J_1) + J_i)}{(1 + 2(\hat{\sigma}_1(J_1) + N))(1 + 2(J_i + \hat{\sigma}_1(J_1)))} \right]$$

$$- (N-1)(N - J_i - 1) \frac{2}{(1 + 2(\hat{\sigma}_1(J_1) + N))^2} - \frac{2(N-1)\left[1 + 2(\hat{\sigma}_1(J_1) + J_i + 1)\right]^2}{(1 + 2(\hat{\sigma}_1(J_1) + N))^2(1 + 2(J_i + \hat{\sigma}_1(J_1)))^2} \left[2(\hat{\sigma}_1(J_1) + 1)^2 + J_i - 1\right]$$

$$- (N-1) \frac{[1 + (\hat{\sigma}_0(J_1) + J_i)(\hat{\sigma}_0(J_1) + N)]}{(1 + \hat{\sigma}_0(J_1) + N)(1 + J_i + \hat{\sigma}_0(J_1))} \left(\frac{1 + 4\hat{\sigma}_0(J_1) + 2(J_i + N) + (\hat{\sigma}_0(J_1) + J_i)(\hat{\sigma}_0(J_1) + N)}{2(1 + \hat{\sigma}_0(J_1) + N)(1 + J_i + \hat{\sigma}_0(J_1))} \right)$$

$$+ (N-1)(N - J_i - 1) \frac{1}{2(1 + \hat{\sigma}_0(J_1) + N)^2} + \frac{(N-1)\left(2 + \hat{\sigma}_0(J_1) + J_i\right)^2}{2(1 + \hat{\sigma}_0(J_1) + N)^2(1 + J_i + \hat{\sigma}_0(J_1))^2} \left[(\hat{\sigma}_0(J_1) + 1)^2 + J_i - 1\right].$$

For the calculation of $\hat{K}(J)$, I make use of Matlab (see the code at the end). The following table gives the buyer's efficient and equilibrium investment threshold for any $J \subset N$, and $N = 10$ sellers.

K^*	0.5978
$\hat{K}(9)$	0.479314
$\hat{K}(8)$	0.482745
$\hat{K}(7)$	0.487170
$\hat{K}(6)$	0.493026
$\hat{K}(5)$	0.501004
$\hat{K}(4)$	0.512164
$\hat{K}(3)$	0.527778
$\hat{K}(2)$	0.545794
$\hat{K}(1)$	0.582257

3. Welfare

For the calculations of welfare, the efficient welfare is given by $W^*(b^*, \sigma_b^*) = TS^*(b^*, \sigma_b^*) - kb^* - \psi(\sigma_b^*)$, and depending on the efficient buyer's investment decision (invest or not invest), efficient welfare is equal to:

$$\begin{aligned} W^*(1, \sigma_1^*) &= \frac{2(\sigma_1^* + N)}{1 + 2(\sigma_1^* + N)} - k - (\sigma_1^*)^2/2, \\ W^*(0, \sigma_0^*) &= \frac{(\sigma_0^* + N)}{2 + 2(\sigma_0^* + N)} - (\sigma_0^*)^2/2. \end{aligned}$$

For any other equilibrium, the welfare generated depends on the equilibrium investment $(\hat{b}(J), \hat{\sigma}_b(J_1))$, therefore $\hat{W}(\hat{b}(J), \hat{\sigma}_b(J_1)) = TS^*(\hat{b}(J), \hat{\sigma}_b(J_1)) - k\hat{b}(J) - \psi(\hat{\sigma}_b(J_1))$. Therefore,

$$\begin{aligned} \hat{W}(1, \hat{\sigma}_1(J_1)) &= \frac{2(\hat{\sigma}_1(J_1) + N)}{1 + 2(\hat{\sigma}_1(J_1) + N)} - k - (\hat{\sigma}_1(J_1))^2/2, \\ \hat{W}(0, \hat{\sigma}_0(J_1)) &= \frac{2(\hat{\sigma}_0(J_1) + N)}{2 + 2(\hat{\sigma}_0(J_1) + N)} - (\hat{\sigma}_0(J_1))^2/2 \end{aligned}$$

For the calculation of $\hat{W}(J)$, I make use of Python (see the code at the end). The following table gives the values of the efficient welfare and the equilibrium welfare for any $J_1 \subset N$, and $N = 10$ sellers.

	$b=1$	$b=0$
$W^*(b,\sigma_b^*)$	0.5023877	0.4545539
$\hat{W}(b,\hat{\sigma}_b(9))$	0.5023877	0.4545539
$\hat{W}(b,\hat{\sigma}_b(8))$	0.5023744	0.4545538
$\hat{W}(b,\hat{\sigma}_b(7))$	0.5023451	0.4545530
$\hat{W}(b,\hat{\sigma}_b(6))$	0.5022892	0.4545506
$\hat{W}(b,\hat{\sigma}_b(5))$	0.5021875	0.4545453
$\hat{W}(b,\hat{\sigma}_b(4))$	0.5020029	0.4545344
$\hat{W}(b,\hat{\sigma}_b(3))$	0.5016593	0.4545116
$\hat{W}(b,\hat{\sigma}_b(2))$	0.5009807	0.4544610
$\hat{W}(b,\hat{\sigma}_b(1))$	0.4994978	0.4543311

Matlab and Python Codes

4. Matlab code for the solution of the seller's equilibrium investment

```
NumIter = 500;
Tolerance = 0.0000001;
InitVal = 0.02;

N = 10;
for J1 = 1:9
    b = 0;
    SigmaRightFcn = @(SigmaVar)(b + 1)^2*(1 + (b + 1)*J1 + ...
        (2*(b + 1)^2)*((N - (J1 + 1))*(SigmaVar + 1) ) )/...
        (2*(1 + (b + 1)+J1)*(1 + (b + 1)*(SigmaVar + N))^2);
    SigmaRightVal = FixedPointFcn(SigmaRightFcn, InitVal, Tolerance, ...
        NumIter);
    OutResults(J1,1:2) = [J1 , SigmaRightVal];
    b = 1;
    SigmaRightFcn = @(SigmaVar)(b + 1)^2*(1 + (b + 1)*J1 + ...
        (2*(b + 1)^2)*((N - (J1 + 1))*(SigmaVar + 1) ) )/ ...
        (2*(1 + (b + 1)+J1)*(1 + (b + 1)*(SigmaVar + N))^2);
    SigmaRightVal = FixedPointFcn(SigmaRightFcn, InitVal, Tolerance, ...
        NumIter);
    OutResults(J1,3) = SigmaRightVal;
end
tablatex(flip(OutResults,1));

function Value = FixedPointFcn(InFunction, x, Epsilon, Iter)
    i = 1;
    y = feval(InFunction, x);
    while abs(y - x) > Epsilon && i + 1 < Iter
        i = i + 1;
        x = y;
```

```
        y = feval(InFunction , x);
    end
    Value = y;
end

function tablatex(matrix)
    fid = fopen('table.tex','w');
    fprintf(fid,'\\documentclass{article}\n');
    fprintf(fid,'\\begin{document}\n');
    fprintf(fid,' \\begin{tabular}{ |');

    for col=1:size(matrix,2)
        fprintf(fid,'l | ');
    end
    fprintf(fid,'}\n\\hline\n');

    % now write the elements of the matrix
    for r=1:size(matrix,1)
        for c=1:size(matrix,2)
            if c==size(matrix,2)
                fprintf(fid,'%f',matrix(r,c));
            else
                if(c == 1)
                    fprintf(fid,'$\\hat{\\sigma}(J_1= %d )$& ', ...
                        matrix(r,c));
                else
                    fprintf(fid,'%f & ',matrix(r,c));
                end
            end
        end

        fprintf(fid,' \\\\ \\hline \n');
    end
    fprintf(fid,'\\hline\n');
    fprintf(fid,'\\end{tabular}\n');
    fprintf(fid,'\\end{document}\n');
    fclose(fid);
 return
```

```
end
```

5. Matlab code for the solution of the buyer's equilibrium investment

```
NumIter = 500;
Tolerance = 0.0000001;
InitVal = 0.02;

N = 10;
for J1 = 1:9
    b = 0;
    SigmaRightFcn = @(SigmaVar)(b + 1)^2*(1 + (b + 1)*J1 + ...
        (2*(b + 1)^2)*((N - (J1 + 1))*(SigmaVar + 1) ) )/...
        (2*(1 + (b + 1)+J1)*(1 + (b + 1)*(SigmaVar + N))^2);
    SigmaRightVal = FixedPointFcn(SigmaRightFcn, InitVal, Tolerance, ...
        NumIter);
    OutResults(J1,1:2) = [J1 , SigmaRightVal];
    Sigma0 = SigmaRightVal;
    b = 1;
    SigmaRightFcn = @(SigmaVar)(b + 1)^2*(1 + (b + 1)*J1 + ...
        (2*(b + 1)^2)*((N - (J1 + 1))*(SigmaVar + 1) ) )/ ...
        (2*(1 + (b + 1)+J1)*(1 + (b + 1)*(SigmaVar + N))^2);
    SigmaRightVal = FixedPointFcn(SigmaRightFcn, InitVal, Tolerance, ...
        NumIter);
    OutResults(J1,3) = SigmaRightVal;
    Sigma1 = SigmaRightVal;
    if(J1 == 9)
        KStarT1 = (2*(Sigma1 + N))/((1 + 2*Sigma1 + 2*N));
        KStarT2 = (Sigma0 + N)/(2*(1 + Sigma0 + N));
        KStarT3 = (1/2)*(Sigma1 + Sigma0)*(Sigma1 - Sigma0);
        KStar = KStarT1 - KStarT2 - KStarT3;
    end
    KJTerm1 = -(N - 1)*(2*(Sigma1 + N))/(1 + 2*(Sigma1 + N));
    KJTerm2 = (N - 1)*(Sigma0 + N)/(2*(1 + Sigma0 + N));
    J = J1;
    Denom = (1 + 2*(Sigma1 + N))*(1 + 2*J);
```

```
    Denom = Denom*Denom;
    Term1 = 4*(N + 2*J*(N + Sigma1) - 1)*(2*(1 + J)*(1 + Sigma1) + ...
        N*(1 + 2*J));
    Term1 = Term1/Denom;
    Term2 = 2*(N - J - 1)/((1 + 2*(Sigma1 + N))^2);
    Term3 = J*2*(1 + 2*(Sigma1 + J + 1))^2;
    Term3 = Term3/Denom;
    KJTerm3 = Term1 - Term2 - Term3;

    Denom = (1 + Sigma0 + N)*(1 + J);
    Denom = 2*Denom*Denom;
    Term1 = (N + J*(N + Sigma0) - 1)*(1 + Sigma0 + (2 + Sigma0 + N)* ...
        (1 + J));
    Term1 = Term1/Denom;
    Term2 = N - J - 1 + J*((2 + Sigma0 + J)^2)/((1 + J)^2);
    Term2 = Term2/(2*(1 + Sigma0 + N)^2);
    KJTerm4 = (-1)*Term1 + Term2;

    Denom = (1 + 2*(Sigma1 + N))*(1 + 2*(J + Sigma1));
    Denom = Denom*Denom;
    Term1 = (N - 1)*4*(1 + 2*(Sigma1 + J)*(Sigma1 + N))*(2*(Sigma1 + J) ...
        + 2*(Sigma1 + N)*(1 + Sigma1 + J));
    Term1 = Term1/Denom;
    Term2 = (N - 1)*(N - J - 1)*2/((1 + 2*(Sigma1 + N))^2);
    Term3 = 2*(N - 1)*((1 + 2*(Sigma1 + J +1))^2)*(2*(Sigma1 + 1)^2 + ...
        J - 1);
    Term3 = Term3/Denom;
    KJTerm5 = Term1 - Term2 - Term3;

    Denom = (1 + Sigma0 + N)*(1 + J + Sigma0);
    Denom = 2*Denom*Denom;
    Term1 = (N - 1)*(1 + (Sigma0 + J)*(Sigma0 + N))*(1 + 4*Sigma0 + ...
        2*(J + N) + (Sigma0 + J)*(Sigma0 + N));
    Term1 = Term1/Denom;
    Term2 = (N - 1)*(N - J - 1)/(2*((1 + Sigma0 + N)^2));
    Term3 = (N - 1)*((2 + Sigma0 + J)^2)*((Sigma0 + 1)^2 + J - 1);
    Term3 = Term3/Denom;
    KJTerm6 = (-1)*Term1 + Term2 + Term3;
```

```
    KJ = KJTerm1 + KJTerm2 + KJTerm3 + KJTerm4 + KJTerm5 + KJTerm6;

    OutResults(J1,4) = KJ;
end

tablatex(flip(OutResults,1));

function Value = FixedPointFcn(InFunction, x, Epsilon, Iter)
    i = 1;
    y = feval(InFunction, x);
    while abs(y - x) > Epsilon && i + 1 < Iter
        i = i + 1;
        x = y;
        y = feval(InFunction , x);
    end
    Value = y;
end

function tablatex(matrix)
    fid = fopen('table.tex','w');
    fprintf(fid,'\\documentclass{article}\n');
    fprintf(fid,'\\begin{document}\n');
    fprintf(fid,' \\begin{tabular}{ |');

    for col=1:size(matrix,2)
        fprintf(fid,'l | ');
    end
    fprintf(fid,'}\n\\hline\n');

    % now write the elements of the matrix
    for r=1:size(matrix,1)
        for c=1:size(matrix,2)
            if c==size(matrix,2)
                fprintf(fid,'%f',matrix(r,c));
            else
                if(c == 1)
                    fprintf(fid,'$\\hat{\\sigma}(J_1= %d )$& ', ...
                        matrix(r,c));
```

```
                    else
                        fprintf(fid,'%f & ',matrix(r,c));
                    end
                end
            end

            fprintf(fid,' \\\\ \\hline \n');
        end
        fprintf(fid,'\\hline\n');
        fprintf(fid,'\\end{tabular}\n');
        fprintf(fid,'\\end{document}\n');
        fclose(fid);
    return
end
```

6. Python code for the efficient and equilibrium welfare

```
import math

x=[0.007171,0.007171,0.010321,0.014123,0.018804,0.024708,0.032385,0.042775,
0.057621,0.080569]

print("Buyer invest")
for i in range(10):
  y=2*(x[i]+10)/(1+2*(x[i]+10))-0.45-(x[i]**2)/2
  print(y)
print("Buyer does not invest")
X=[0.003754,0.003754,0.004545,0.005514,0.006727,0.008291,0.010382,0.013323,
0.017761,0.025234]

for i in range(10):
  y=(x[i]+10)/(2+2*(x[i]+10))-(x[i]**2)/2
  print(y)
```